Extracting information from interimpact intervals in a mechanical oscillator

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Monitoring all the state variables in dynamic experiments may be difficult or even impossible. It is also desirable to reduce the coupling between the system under study and the measuring device to as low a level as possible. To these ends, we investigated the use of interimpact interval as a discrete state variable. It is well established that topological information can be obtained from delay coordinate embedding and thus not all of the state variables, or even a continuous set of a single variable, need to be measured. In the case of impacting systems, the impacts can be viewed as discrete events that can then be used to reconstruct more general features of the behavior. The success of such reconstruction techniques will be assessed in this paper. [S1063-651X(97)05309-9]

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I. INTRODUCTION

Examples of impacting systems can be found throughout the mechanical world: these include a gear with backlash, a ship moored to a fender, an offshore platform with anchoring lines that are alternately slack and taut, automotive parts that become unbalanced, and loosely connected structures in general. Generic impacting oscillators can provide valuable insight into the behavior of these various applied systems.

Much of the previous work in impacting systems has involved piecewise linear oscillators, which obey Hooke's law above and below the impact point, but have a discontinuity in the force-displacement relationship. This discontinuity in an otherwise linear system allows for the manifestation of the characteristics of nonlinear systems, including period-doubling bifurcations and chaos [1–8].

Traditionally, experiments are constructed so that a continuous measurement of the state of the system can be made. However, in many instances in more complicated mechanical systems, obtaining a continuous measurement of the state of the system may not be feasible. Also, directly attaching a measuring device to the system often disrupts the system, altering system characteristics, such as damping. This Brief Report assesses noninvasive techniques for determining information about the state of the system, only measuring a subset of behavior. By measuring the time between each occurrence of impact, we will construct a one-dimensional map of the system's dynamics. From this discrete map, we infer more general characteristics of the continuous system. The idea of using an interspike interval as a means of extracting information from a dynamical system has been proposed by Sauer [9,10] and since, for example, many applications in physiology involve sudden activation of electrical potentials it is natural that early uses of this technique are to be found in the field of biomedical research [11].

Two noninvasive techniques will be examined. A piezoelectric membrane placed on the reverse side of the impact surface and a microphone will be used to record the sound of impact. These two methods will be compared with experimental results obtained using a rotational potentiometer placed at the pivot, which was used for continuous state variable measurements in previous studies [5], and with computer simulation results obtained from numerical integration of the equations of motion.

II. SYSTEM DEFINITION

This Brief Report focuses on the impacting one-degree-of-freedom rigid-arm pendulum. As shown in Fig. 1, this is a pendulum that is subjected to a rigid impact condition at $\theta=0$. The pendulum is constructed in such a way that the natural frequency can be altered by inclining the entire assembly by some angle Φ. This leads to an effective gravitational acceleration $g_e=g\cos\Phi$ and d is a dynamic horizontal base displacement.

The equation of motion is

$$-\frac{\ddot{d}}{L}\cos\,\theta=\ddot{\theta}+\left[\frac{1}{m}+\frac{g_e}{L}\right]\sin\theta. \tag{1}$$

For this system, we impose a base excitation of the form

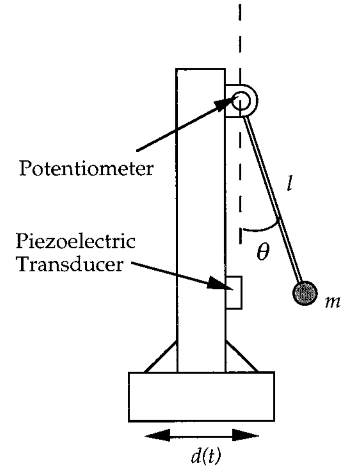

FIG. 1. Schematic diagram of impacting pendulum system.

$$d(t)=A\ \sin\omega t. \tag{2}$$

Adding a viscous damping term to model energy dissipation and nondimensionalizing the equations, we obtain the equation of motion

$$\theta'' + \frac{2\beta}{\eta}\ \theta' + \frac{1}{4\eta^2}\sin\theta = \frac{A}{L}\cos\theta\ \sin\tau, \tag{3}$$

where β is the nondimensional damping ratio fitted from experiments, η is the frequency ratio ω/ω_0 (the natural frequency of the impacting system ω_0 is twice the frequency of the nonimpacting pendulum of the same length $\omega_n = \sqrt{g_e/L}$), and τ is the scaled time ωt.

The impact condition is incorporated by assuming a purely elastic collision. In this study, as in some previous ones, the coefficient of restitution was defined as $r=1$ since all forms of energy dissipation are being accounted for in the linear viscous damping term.

The equation of motion was solved using a fourth-order Runge-Kutta algorithm. Special care was taken to monitor the time of impact. Since the contact occurs at equilibrium, the impact is usually quite distinct and low-velocity grazing bifurcations are not encountered [7,12]. The results from the computer simulation of the equations of motion will be compared with the three experimental methods.

III. EXPERIMENTAL PROCEDURE

The pendulum is constructed of a steel ball of diameter 25.4 mm (1 in.), mounted on an aluminum shaft of length 305 mm (12 in.). The shaft is allowed to pivot by means of two shielded bearings connected by a transverse rod. The entire assembly is capable of being tilted at an angle, so that the natural frequency of the pendulum may be changed by altering the effective gravitational acceleration. For the purposes of this experiment, it was found that an inclination of 1.33 rad provided a suitable natural frequency in relation to the frequency capacity of the shake table [5].

The pendulum assembly is attached by clamps to a shake table, which provides the excitation to the system. Forcing is provided by a scotch yoke mechanism that converts the rotational motion of a variable-speed motor to sinusoidal lateral motion of two parallel rails mounted on linear bearings. A stepper motor mounted on the yoke allows for control of the amplitude of excitation. Both control of forcing and data acquisition are achieved through the Macintosh-based National Instruments LabVIEW system and using a National Instruments NB-MIO-16 data acquisition interface board.

As discussed in the Introduction, we are interested in noninvasive methods of data acquisition. These techniques are desirable for two reasons. First, information about the dynamical behavior of the system may be gathered in cases where it is not possible to directly attach a potentiometer to the moving part. Second, a nonintrusive method of data collection will uncouple the system from the measurement device, eliminating any additional damping, for example. In this study, two nonintrusive techniques, utilizing a microphone and a piezoelectric transducer, were used and compared with experimental results obtained by a potentiometer and with numerical simulation.

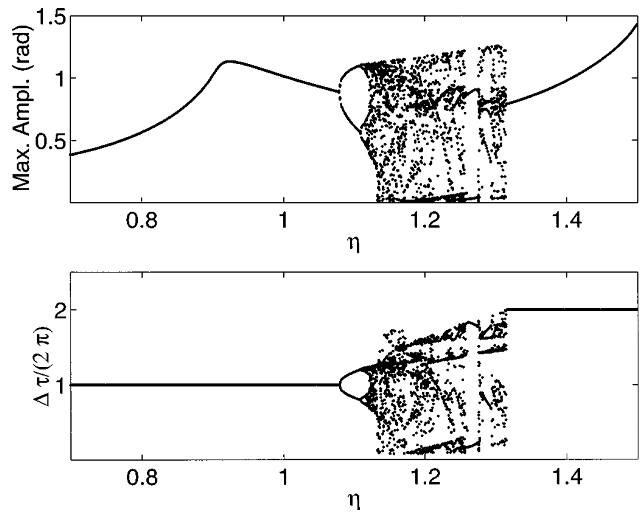

FIG. 2. Response of the impacting pendulum plotted as a function of the nondimensional frequency ratio η based on numerical simulation: (a) a conventional amplitude response diagram and (b) an interimpact interval, nondimensionalized with respect to the forcing period.

In previous work [5], the displacement of the pendulum has been measured by a rotational potentiometer attached at the pivot point. This method will be used as a base line for comparison with the alternative experimental techniques. Since the pendulum was not allowed to rotate over angles greater than 90°, a one-turn potentiometer (Spectrol model 132-00-00-103) was used. This allows for greater sensitivity over the range of motion. The potentiometer was easily calibrated through a range of static displacements and a linear voltage-displacement expression can be determined.

One alternative method of taking data involves the use of a piezoelectric strain gauge. The strain gauge was mounted on the underside of the impact surface. The transducer itself was constructed of polyvinylidene fluoride (PVDF), a thin reflective film that is readily applied to any surface through the use of double-sided tape or similar adhesive.

When the PVDF membrane is bent, it creates a potential difference across the two contacts. The raw trace of voltage versus time was read into the LabVIEW system and post-processing of the data was carried out in order to find the locus of maxima that indicated the occurrence of an impact. The time of occurrence of each peak was saved and a one-dimensional mapping of time intervals between impacts was obtained. The second alternative method of data acquisition was the direct recording of the sound produced by the impact between the steel ball and the impact plate and again the time of each impact was determined.

A typical numerically generated bifurcation diagram is shown in Fig. 2. Figure 2(a) shows a conventional amplitude response diagram as a function of frequency ratio. For frequency ratios below 1.1 the response is similar to a standard (rectified) linear oscillator, although the resonant peak occurs just below the natural frequency because of the slight softening spring effect at these amplitudes. However, a period-doubling cascade and chaotic window appear between $\eta = 1.1$ and 1.3 and for higher frequencies a period-two (alternating impact) oscillation occurs. Figure 2(b) shows the same data but with the interimpact interval as the response

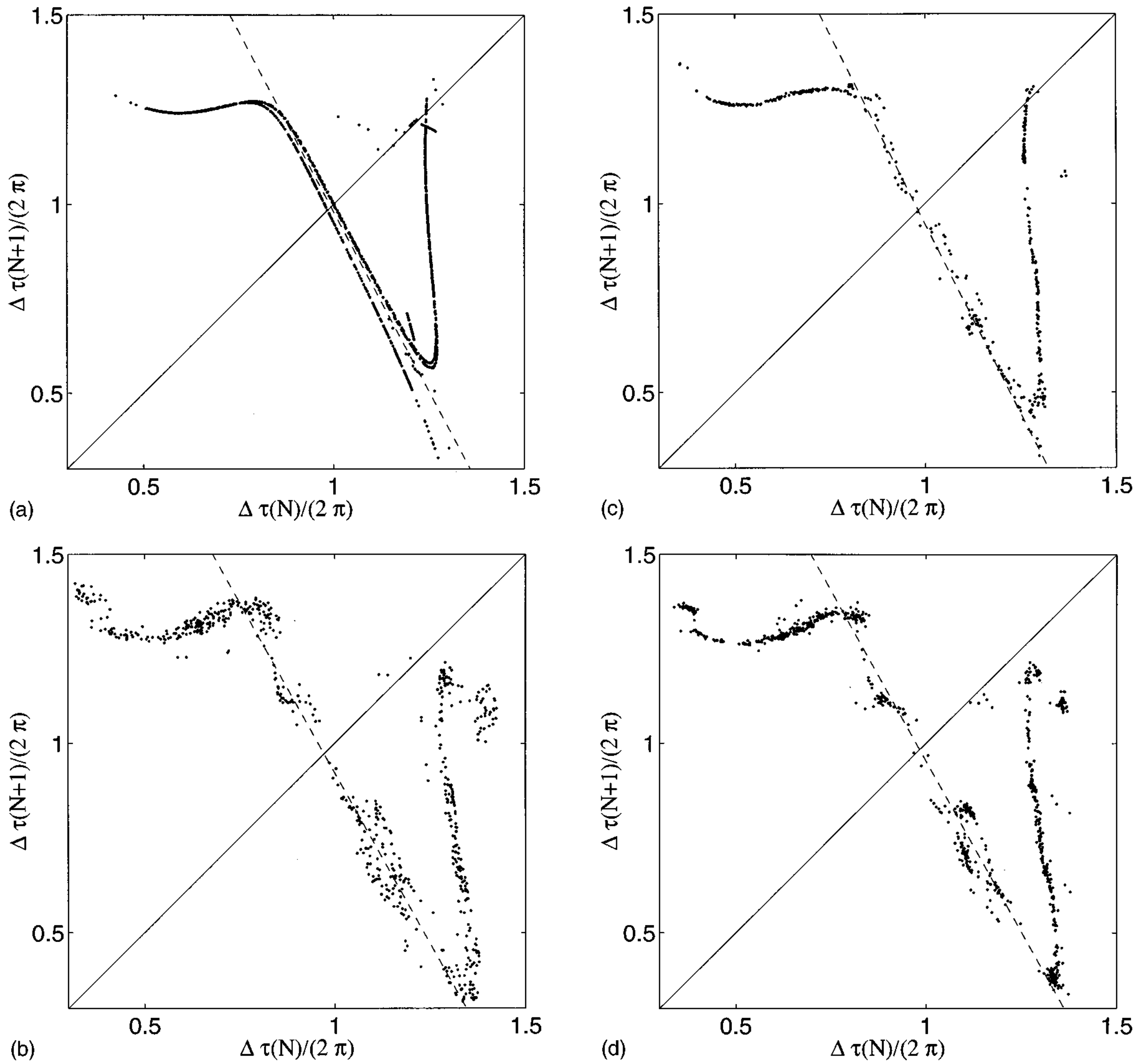

FIG. 3. Chaotic attractor ($\eta=1.12$), characterized by using time-lag embedding with the interimpact interval as the measured variable: (a) simulation, (b) potentiometer, (c) PVDF, and (d) audio.

measure. The constant time between impacts corresponding to the period-one attractor is clear. In the following section we take a closer look at two specific parameter values in the period-one and chaotic regimes.

IV. RESULTS

For purposes of comparison, the amplitude of forcing will be kept fixed at 50.8 mm (2 in.) center to peak and the frequency will be varied as the control parameter. The results of a measurement of the free-decay characteristics of the system showed that the damping ratio $\beta\approx 0.07$. Also from the unforced oscillation of the system, we can determine the effective length of the pendulum from knowledge of the natural frequency and the modified gravitational acceleration. This effective length was calculated as 225 mm.

First, we examine the behavior of the system in a period-one regime. At a forcing frequency of 0.9 Hz ($\eta=0.87$), the system exhibits period-one oscillations. The resulting map should be one point that lies on the $\Delta\tau(N+1)=\Delta\tau(N)$ line, i.e., the time delay between successive impacts is a constant. Typically, the fixed point associated with the period-one attractor showed a remarkably similar mean location regardless of the measurement technique. However, the *scatter* based on a simple standard deviation calculation, which in all cases was small, varied according to the method used: smallest for audio recording ($s_x=6\times 10^{-4}$), followed by the PVDF sensor ($s_x=1.4\times 10^{-3}$), and the potentiometer ($s_x=3.1\times 10^{-3}$).

As we increase the forcing frequency further, the system enters a region of chaotic oscillations. At a frequency of 1.15 Hz ($\eta=1.12$), the map of interimpact times takes on the form as seen in Fig. 3: a complex, asymmetrical curve [13]. The curve produced by the PVDF sensor generated the results that most closely correlated with those of the simulation. While the audio recording performed well for simple periodic behavior, the quality of results decreased as the system behavior became more complex. This phenomenon is probably related to the fact that the sound of the period-one oscillation was somewhat louder and more distinct than those of the other types of behavior. Another disadvantage to the audio recording method is that it requires a high sam-

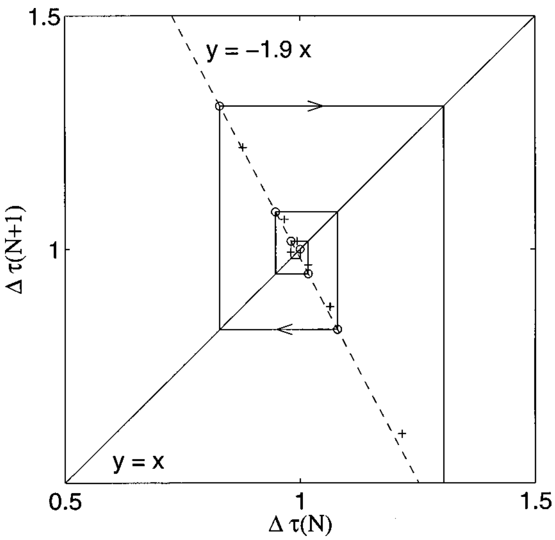

FIG. 4. Comparison of numerical simulation (+) and iterated map (○) for an unstable period-one orbit embedded in the chaotic attractor shown in Fig. 3.

pling rate, typically 11.025 kHz, leading to large data files and high memory requirements. The PVDF sensor, on the other hand, produced good results in both periodic and chaotic regimes and required only a slightly faster sampling rate than the potentiometer, 1.25 kHz as compared to 1 kHz.

From the data produced by the one-dimensional map of chaotic impacting, we can evaluate the stability of the underlying (unstable) period-one behavior of the map. This is accomplished by fitting a line to the map at its intersection with the $\Delta\tau(N+1)=\Delta\tau(N)$ line. As seen in Fig. 3, in all cases, the slope m of the resulting line was less than -1, indicating that the fixed point is unstable and diverges in a typical ''flipping'' mode.

The instability of the fixed point can be seen by comparing the performance of the simulation in the region near the fixed point with that of an analytical one-dimensional map derived from the linear fit to the simulation in the range $0.9<x<1.2$. From the linear fit, we can approximate the behavior of the map in the region of the period-one fixed point $\Delta\tau=2\pi$ by

$$x_{i+1}\approx -1.9x_i, \tag{4}$$

where x is the distance from the fixed point, measured along the fitted line. Figure 4 shows a typical comparison between the simulation and the evolution of the linearized map.

V. CONCLUSION

This brief study compares results from two nonintrusive data acquisition techniques with results obtained from continuous time series from a potentiometer and results from numerical simulation. Using one-dimensional maps generated from the duration between impacts, it was possible to gain insight into the behavior of the system without using continuous data. The results from the PVDF sensor produced good agreement with numerical simulation at a low data processing cost. Interimpact interval analysis is a viable dynamic reconstruction approach, provided, of course, impact occurs.

ACKNOWLEDGMENTS

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